Production of vortices in the dual Ginzburg–Landau theory

2002 J. Phys. A: Math. Gen. 35 10739

(http://iopscience.iop.org/0305-4470/35/50/305)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **35** (2002) 10739–10746 PII: S0305-4470(02)38627-X

Production of vortices in the dual Ginzburg–Landau theory

T Dobrowolski

Institute of Physics AP, Podchorążych 2, 30-084 Cracow, Poland

Received 24 June 2002, in final form 22 October 2002
Published 4 December 2002
Online at stacks.iop.org/JPhysA/35/10739

Abstract

The number density of vortices produced during a quench in the Abelian Higgs model is predicted. Assuming that the dual Landau–Ginzburg model provides an effective infrared description of QCD, the estimation of the number density of mesons and glueballs produced during the phase transition to confined phase is predicted.

PACS numbers: 11.27.+d, 05.70.Fh, 12.38.Aw, 12.39.Mk

1. Introduction

A well-known property of QCD on short distances is asymptotic freedom. However, the properties of the theory at large distances are problematic yet. We know that neither a free quark nor a free gluon has ever been observed in nature. Upper experimental limits have been established for the production of quarks in high-energy collisions. Since the early days of QCD it has been suggested that the mechanism by which colour is confined could be dual superconductivity of second type. This idea was inspired by pioneering works of 't Hooft and Mandelstam [1] who argued that the quark–antiquark interaction force can be produced by flux tubes of Abrikosov type [2]. The flux tubes could be produced by a dual Meissner effect squeezing the chromoelectric field acting between quark and antiquark into a flux tube. This mechanism makes the energy proportional to the length of the tube. Recent lattice simulations lend some support to the hypothesis of dual superconductivity as the mechanism of confinement in QCD [3–5]. There also exist suggestions that monopole condensation is the source of both the confinement and the chiral symmetry breaking [6].

Although there are a lot of doubts concerning the dual description of QCD, it is believed that the dual description should also be a gauge theory, possibly with interchange of the role of magnetic and electric fields. There also exist results suggesting the relevance of the degrees of freedom identified by the maximal Abelian projection for description of the confinement [7, 8]. In this context the effects from maximal Abelian gauge Gribov copies can also be taken into account [9]. In addition, it is expected that the strong coupling regime of the original model is mapped in the weak coupling regime of the dual model [5].

In the nonperturbative regime the QCD under assumption of Abelian dominance can be reduced to the dual $U^2(1)$ Ginzburg–Landau effective theory [10]. The purpose of this paper is to estimate the number density of the flux tubes created during the transition from quark–gluon plasma to confined phase. Probably, the matter in the deconfined phase is produced at present at the RHIC. The energy densities reached in Brookhaven National Laboratory are 10 to 100 times higher than that of nuclear matter [11]. The energy density at this level is probably naturally produced in the cores of neutron stars and probably was present at the early stages of evolution of the Universe. This is a reason why results of the RHIC can shed new light on the big bang scenario. The number density of topological defects produced during the second-order phase transition does not depend on details of the model and it is mainly determined by the Kibble–Zurek mechanism. This is a reason why the number density of produced flux tubes is solely determined by the Kibble–Zurek critical exponent [12] and in this sense it does not depend on the details of the infrared effective model. The structure of QCD vacuum, in the dual Ginzburg–Landau model, seems to be similar to the structure of the superconductor. The type of superconductivity is still under dispute [13, 14].

If we assume that we have superconductivity of the second type then colour confinement is realized by the electric flux tubes which are the topological solutions of the model. The mesons in this scenario are just flux tubes terminated by the quark–antiquark pair, and glueballs are proposed to be the flux-tube rings [15].

The existence of colourless states made only of gauge fields is a natural consequence of self-interaction of gauge fields in QCD. The glueball solutions of the dual Ginzburg–Landau model are classically unstable and they decay in finite time [16].

The situation is similar to the lack of stability of the classical hydrogen atom. We know that on the quantum level its size is stabilized and its radius is of order 10^5 fm. Using this analogy Koma *et al* suggested a quantum model describing the flux-tube ring. The quantum radius of this structure does not exceed 1 fm and is approximated as $R_G \approx 0.25$ fm. Later in this paper this approximation is used as a constraint on the size of the flux-tube loop.

In the next section the dual Ginzburg–Landau model is recalled and the number density of produced glueballs is calculated. The last section contains remarks.

2. Vortex production in the weak coupling limit of the Abelian Higgs model

The effective low-energy Lagrangian consists of dual gauge fields $\vec{B}_\mu = \left(B_\mu^3, B_\mu^8\right)$ and three complex scalar monopole fields (χ^α) [17]. This Lagrangian in the colour singlet sector reduces to the dual, $U(1)$ invariant Abelian Higgs model [18, 13]

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\chi)^*(D^\mu\chi) - \hat{\lambda}(\chi^*\chi - \hat{v}^2)^2 \tag{1}$$

where B_μ are rescaled dual gauge fields and χ is the complex scalar monopole field. The field strength

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2}$$

and covariant derivatives have typical form

$$D_\mu\chi = \partial_\mu\chi + \mathrm{i}\hat{g}B_\mu\chi. \tag{3}$$

The Euler–Lagrange equations for the Lagrangian (1)

$$D_\mu D^\mu\chi = 2\hat{\lambda}\chi(\hat{v}^2 - \chi^*\chi) \tag{4}$$

$$\partial^\nu F_{\mu\nu} = \mathrm{i}\hat{g}(\chi\partial_\mu\chi^* - \chi^*\partial_\mu\chi) + 2\hat{g}^2B_\mu\chi\chi^*. \tag{5}$$

These equations, depending on the parameters of the model, describe type I or type II superconductivity. The first type dual superconductivity appears if the self-interaction of the monopole scalar field is stronger than its coupling to the dual gauge field, i.e. $2\hat{\lambda} < \hat{g}^2$. However, in the opposite case, i.e. for the weak coupling $\hat{g}$, the superconductor is punched by the Abrikosov flux tubes. Let us consider weak $\hat{g}$ coupling sector of the dual model. In this regime the vacuum has in a natural way a structure of the second type superconductor. The most important fact is that this is a case of strong coupling of the original model, i.e. QCD. A gauge field in this regime decouples and we can take into consideration only the scalar sector of the model:

$$\partial_t^2\chi - \Delta\chi = 2\hat{\lambda}\chi(\hat{v}^2 - \chi^*\chi). \tag{6}$$

To provide inhomogenous initial conditions we introduce to the equation of motion the white Gaussian noise η. To stabilize the system against external perturbations we have to also add the damping therm $\gamma\partial_t\chi$. Although both terms have simple physical meaning (temperature noise and dissipation) they are treated as auxiliary and they will be excluded by taking $\gamma \to 0$ limit at the end of the calculation

$$\partial_t^2\chi + \gamma\partial_t\chi - \Delta\chi = 2\hat{\lambda}\chi(\hat{v}^2 - \chi^*\chi) + \eta. \tag{7}$$

The Gaussian white noise is defined by the cumulants

$$\langle\widetilde{\eta}(t,\vec{k})\rangle = 0 \qquad \langle\widetilde{\eta}^*(t,\vec{k})\widetilde{\eta}(t',\vec{k}')\rangle = \frac{2\pi\gamma}{\beta}\delta^{(2)}(\vec{k}-\vec{k}')\delta(t-t') \tag{8}$$

where $\widetilde{\eta}$ is a Fourier transform of the noise $\eta(t,\vec{x}) = \int_{-\infty}^{\infty}\mathrm{d}^2k\,\mathrm{e}^{\mathrm{i}\vec{k}\vec{x}}\widetilde{\eta}(t,\vec{k})$. We assume isotropy and homogeneity of the system and then we consider a section of the system by an arbitrary chosen plane. In the next step we consider the phase transition from quark–gluon plasma to confined phase, i.e. the transition at $T = T_{\text{conf}}$ temperature. Finally we calculate the number density of vortices and antivortices produced on this plane. These vortex structures are traces of the open (mesons) and closed flux tubes (glueballs) created during the phase transition.

We consider a spatially homogenous transition caused by the linear time dependence of the relative temperature $\hat{v}^2 = \hat{v}_0^2\frac{T_{\text{conf}}-T}{T_{\text{conf}}} = \hat{v}_0^2\frac{t}{\tau}$, where 2τ is a quench time. This dependence of the relative temperature on time is a consequence of heating of the system $T(t)$.

To count zeros of the order parameter it is sufficient to use a linear approximation of equation (7). Fourier transformation $\chi(t,x) = \int_{-\infty}^{\infty}\mathrm{d}k\,\mathrm{e}^{\mathrm{i}kx}\widetilde{\chi}(t,k)$ allows for significant simplification of the equation of motion

$$\partial_t^2\widetilde{\chi} + \gamma\partial_t\widetilde{\chi} + \vec{k}^2\widetilde{\chi} = 2\hat{\lambda}\hat{v}_0^2\frac{t}{\tau}\widetilde{\chi} + \widetilde{\eta}. \tag{9}$$

The solutions of the homogeneous equation

$$\widetilde{\chi}_a(t,\vec{k}) = b\,\mathrm{e}^{\frac{1}{2}\gamma z}\sqrt{z}\mathcal{J}_{\frac{1}{3}a}\left(\frac{2\mu}{3}z^{\frac{3}{2}}\right) \tag{10}$$

where $\mathcal{J}_{\frac{1}{3}a}$ is appropriate Bessel function, $\mu^2 = \frac{2\hat{\lambda}\hat{v}_0^2}{\tau}$, $b = \mathrm{e}^{\frac{\gamma^3}{24\mu^2}}$, $z = \frac{1}{\mu^2}\left(\vec{k}^2 - \frac{\gamma^2}{4}\right) - t$ and $(a) = (-1, +1)$ form a basis for construction of the Green function in the model

$$\mathcal{G}(t,t';k) \equiv \mathcal{G}(z,z') = \Gamma\left(\tfrac{2}{3}\right)\Gamma\left(\tfrac{4}{3}\right)\mathrm{e}^{-\frac{\gamma^3}{12\mu^2}}\Theta(z'-z)\,\mathrm{e}^{-\gamma z'}\epsilon^{ab}\widetilde{\chi}_a(z)\widetilde{\chi}_b(z') \tag{11}$$

where ϵ^{ab} is a Levi-Civita symbol equipped in a convention $\epsilon^{-+} = 1$. The solution of equation (9) which satisfies initial conditions $\widetilde{\chi}(t \to -\infty, k) = 0$ and $\partial_t\widetilde{\chi}(t \to -\infty, k) = 0$ is the following:

$$\widetilde{\chi}(t,k) = \int_{-\infty}^{t}\mathrm{d}t'\,\mathcal{G}(t,t';k)\widetilde{\eta}(t',k). \tag{12}$$

The central quantity which allows for estimation of the number density of the vortices produced during the phase transition from normal to superconducting state is the power spectrum of the system. The power spectrum is defined by the equal time field correlator

$$\langle\widetilde{\chi}^*(t,\vec{k})\widetilde{\chi}(t,\vec{k}')\rangle = \mathcal{P}(t,\vec{k})\delta^{(2)}(\vec{k}-\vec{k}') \tag{13}$$

where $\langle\ldots\rangle$ denotes an average over realizations of the noise. This definition leads to the expression

$$\mathcal{P}(t,k) = \frac{2\pi\gamma}{\beta}\int_z^\infty \mathrm{d}z'|\mathcal{G}(z,z')|^2. \tag{14}$$

The explicit form of the power spectrum is rather complicated

$$\mathcal{P}(t,k) = \frac{2\pi\gamma}{\beta}\Gamma^2\left(\frac{2}{3}\right)\Gamma^2\left(\frac{4}{3}\right)\epsilon^{ab}\epsilon^{a'b'}\,\mathrm{e}^{\gamma z}z\mathcal{J}_{\frac{1}{3}b}\left(\frac{2\mu}{3}z^{\frac{3}{2}}\right)\mathcal{J}^*_{\frac{1}{3}b'}\left(\frac{2\mu}{3}z^{\frac{3}{2}}\right)\mathcal{F}_{aa'}(z). \tag{15}$$

In this formula we have summation over indices $(a,b) = (-1,+1)$, and the last quantity is defined by the integral

$$\mathcal{F}_{aa'}(z) = \int_z^\infty \mathrm{d}z'\,z'\mathcal{J}_{\frac{1}{3}a}\left(\frac{2\mu}{3}z^{\frac{3}{2}}\right)\mathcal{J}^*_{\frac{1}{3}a'}\left(\frac{2\mu}{3}z'^{\frac{3}{2}}\right)\mathrm{e}^{-\gamma z'}. \tag{16}$$

This function contains an information about the correlation length which in the $\gamma\to 0$ limit and at freeze-out time is $\xi\approx\left(2\hat{\lambda}\hat{v}_0^2\frac{\hat{t}}{\tau}\right)^{-\frac{1}{2}}$. On the other hand the quench time is related to the correlation length as follows: $\tau\approx\tau_0\left(2\hat{\lambda}\hat{v}_0^2\xi^2\right)^2$. In the Kibble–Zurek scenario of production of the topological defects the freeze-out time $\hat{t}$ is an instant of time when the system regains capacity to respond to the change of external parameters. The correlation length at that time sets the characteristic length scale for the initial defect network.

For sufficiently late times z is negative, and therefore we introduce positive variable $y = \hat{t} - \frac{1}{\mu^2}\left(k^2 + \frac{\gamma^2}{4}\right)$. We also use the relation between Bessel and modified Bessel functions $\mathcal{J}_{\frac{1}{3}a}(-\mathrm{i}x) = \mathrm{e}^{\mathrm{i}a\frac{\pi}{2}}\mathcal{I}_{\frac{1}{3}a}(x)$ and large x behaviour of the modified Bessel functions $\mathcal{I}_{\frac{1}{3}a}(x)$. The final result of this calculation is an explicit expression for the power spectrum

$$\mathcal{P}(\hat{t},k)\approx\mathcal{A}\frac{\exp\left(\frac{4\mu}{3}\left(\hat{t}-\frac{\vec{k}^2}{\mu^2}\right)^{\frac{3}{2}}\right)}{\sqrt{\hat{t}-\frac{\vec{k}^2}{\mu^2}}} \tag{17}$$

where

$$\mathcal{A} = \frac{9\sqrt{\gamma}\Gamma^2\left(\frac{2}{3}\right)\Gamma^2\left(\frac{4}{3}\right)}{2\mu\sqrt{\pi}}.$$

The number density of the produced vortices can be determined with the help of the approximate form of the power spectrum and the Liu–Mazenko–Halperin formula [19]

$$n = \frac{1}{2\pi}\frac{\int_{S_{k_m}}\mathrm{d}^2k\,\vec{k}^2\mathcal{P}(\hat{t},\vec{k})}{\int_{S_{k_m}}\mathrm{d}^2k\,\mathcal{P}(\hat{t},\vec{k})}. \tag{18}$$

The integration in this formula is restricted to the interior of the circle of radius $|\vec{k}_m| = \frac{1}{\xi}$. The cut-off in this formula is indispensable because the zeros are produced on all scales and we know that only those zeros which correspond to unstable modes of the system are able to produce any stable vortex structures. Those zeros are separated at least by the correlation length. The number of zeros on the two-dimensional section in the considered system is determined by the correlation length ξ

$$n\approx 0.1\frac{1}{\xi^2}. \tag{19}$$

It is worth stressing that we counted only zeros which, in course of the evolution, will form the centres of the vortices.

3. Production of particles during the transition to the confined phase

In the zero topological charge sector of the model the number of vortices on the plane is identical to the number of antivortices and therefore the number of vortices which form a basis for estimation of the number of produced mesons and glueballs can be estimated by the formula $n_V = n_{AV} = \frac{1}{2}n \approx 0.05\frac{1}{\xi^2}$.

3.1. Mesons

Having number density of vortex structures on the plane we are also able to estimate the number of open flux tubes contained in the volume of 1 fm^3. The only difference between mesons which correspond to vortices and mesons which correspond to antivortices is their opposite orientation. The number density of mesons is the following:

$$N_M = n\frac{1}{\text{fm}} \approx 0.1\frac{1}{\xi^2}\frac{1}{\text{fm}}. \tag{20}$$

3.2. Glueballs

Now we count the three-dimensional structures of size limited by the closed-loop radius responsible for the existence of zeros on the considered plane. We assume the isotropic distribution of the loops crossing the plane and then estimate the average length of projection of the loop of radius R_G, on the axis perpendicular to the considered plane $\langle l\rangle = 2R_G\sqrt{\langle\cos^2\theta\rangle} \approx 1.4R_G$. This number is determined by the correlation length ξ and the glueball radius as well

$$N_{G+\frac{1}{2}M} = n_V\frac{1}{\langle l\rangle} \approx 0.035\frac{1}{\xi^2 R_G}. \tag{21}$$

To estimate this quantity we used the number of vortices n_V and not the total number of vortex structures n because the trace of each loop on the considered plane is a pair of vortex–antivortex. In addition we obtain half of the meson states.

Finally the number of glueballs produced during the transition can be extracted as follows:

$$N_G = N_{G+\frac{1}{2}M} - \tfrac{1}{2}N_M.$$

On the basis of these results we can also obtain the ratio of the produced mesons to the number of glueballs produced during the phase transition $\frac{N_G}{N_M}$.

3.3. Baryons

One could attempt to also estimate the number density of baryon states produced during the considered phase transition. Using the information gained in single-colour sector we can construct baryon on the basis of the 'three quark picture' and then it corresponds to two zeros being the traces of two flux tubes punching the plane. The number density of baryons is

$$N_B = \frac{1}{2}n_V\frac{1}{\text{fm}} \approx 0.025\frac{1}{\xi^2}\frac{1}{\text{fm}}. \tag{22}$$

We also have to take into account the colour degrees of freedom. The most important from our point of view is the knowledge of possible overlap of flux tubes of different types. To gain this

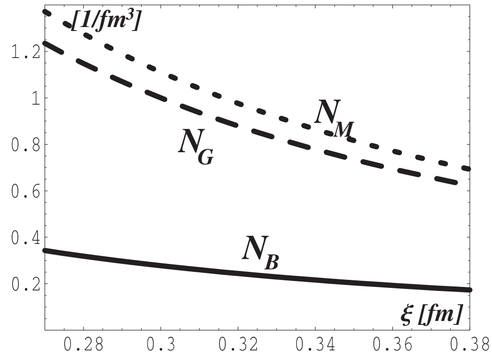

Figure 1. The dependence of the number density of produced particles on the correlation length: meson density (dotted line), glueball density (dashed line) and baryon density (solid line).

information we have to consider the solutions of the complete model [17], but even without this analysis we can expect the additional factor $C \in [1, 3]$ multiplying formula (22). If flux tubes of different types can be located at the same positions then $C = 3$. In the opposite case, i.e. if overlapping is forbidden then $C = 1$. Because we neglected almost a whole baryon internal structure, formula (22) gives the upper bound of the number density of produced baryons.

4. Remarks

At this point it is worth stressing that the results depend on the choice of the 'cut-off' in integrals so that it mainly depends on the quench time. In further considerations we choose the correlation length which gives the number density of produced baryons compatible with that obtained in a completely different way by Ellis *et al*, 0.17–0.35 fm^{-3} [20]. We also adopt the approximation of the radius of the vortex loop given in paper [13], $R_G \approx 0.25$ fm.

The dependence of the particle numbers on correlation length, i.e. on quench time, is demonstrated in figure 1.

According to the chosen strategy the baryon density changes from 0.35 fm^{-3} to 0.17 fm^{-3}. The number density of produced mesons decreases from 1.4 fm^{-3} to 0.68 fm^{-3} and the number density of glueballs decreases from approximately 1.3 fm^{-3} to 0.63 fm^{-3}. The figure was prepared under the assumption of lack of overlapping of the flux tubes of different types. The quench time corresponding to the upper boundary of the considered correlation length interval $\xi = 0.38$ fm is 4.24 times larger than the quench time corresponding to the lower boundary of considered interval $\xi = 0.27$ fm. An important observation is that the ratio of the produced particles does not depend on correlation length at all, and it is approximately equal to 0.9. In the above calculation it was assumed that a very small fraction of the vortex loops forms topologically untrivial knots which can also be experimentally interpreted as excited glueball states (figure 2).

Finally, a question arises about the possibility of experimental verification of the results. It seems that the most favourable conditions are provided during relativistic heavy-ion collisions. Studies in the direction of understanding better the properties of this form of matter are presently performed at the RHIC at Brookhaven National Laboratory. The energy density, estimated in the centre-of-mass frame, attains the level 1–10 GeV fm^{-3} [21], which seems to be sufficient to produce the quark–gluon plasma. Cooling down this system provides the phase transition from deconfined to confined phase which is the subject of the present studies.

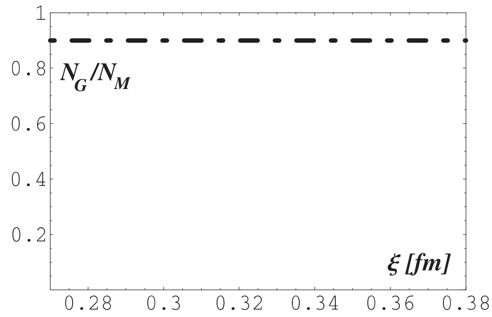

Figure 2. The ratio of the number density of produced glueballs to the number density of produced mesons versus the correlation length.

Acknowledgments

I would like to thank the referees for suggestions which lead to substantial improvement of the paper. I am also indebted to Jacek Dziarmaga for stimulating discussions. This paper is supported in part by ESF COSLAB Programme.

References

[1] 't Hooft G 1981 *Nucl. Phys.* B **190** 455
Mandelstam S 1976 *Phys. Rep.* **23 C** 245
[2] Nielsen H B and Olesen P 1973 *Nucl. Phys.* B **61** 45
Abrikosov A A 1957 *Zh. Eksp. Teor. Fiz.* **32** 1442
[3] Carmona J M, D'Elia M, Del Debbio L, Di Giacomo A, Lucini B and Paffuti G 2002 *Phys. Rev.* D **66** 011503
[4] Di Giacomo A 1999 *Nucl. Phys.* A **661** 13
[5] Kondo Kei-Ichi 1998 *Prog. Theor. Phys. Suppl.* **131** 243
[6] Toki H 1998 *Prog. Theor. Phys. Suppl.* **131** 257
Nambu Y and Jona-Lasinio G 1961 *Phys. Rev.* **122** 345
[7] Suzuki T and Yotsuyanagi I 1990 *Phys. Rev.* D **42** 4257
[8] Kondo Kei-ichi 1998 *Phys. Rev.* D **57** 7467
[9] Bali G S, Bornyakov V, Müller-Preussker M and Schilling K 1996 *Phys. Rev.* D **54** 2863
[10] Kondo K I 1998 *Prog. Theor. Phys. Suppl.* **131** 243
Bali G S, Schlichter C and Schilling K 1998 *Prog. Theor. Phys. Suppl.* **131** 645
Maedan S and Suzuki T 1989 *Prog. Theor. Phys.* **81** 229
[11] McLerran L 2002 *Preprint* hep-ph/0202025 (Invited talk presented at the *Int. Conf. on the Physics and Astrophysics of the Quark Gluon Plasma, Nov. 26–30 2001*)
[12] Dziarmaga J, Laguna P and Zurek W 1999 *Phys. Rev. Lett.* **82** 4749
Karra G and Rivers R J 1997 *Phys. Lett.* B **414** 28
Gill A J and Rivers R J 1995 *Phys. Rev.* D **51** 6949
Zurek W H 1993 *Nature* **317** 505
Zurek W H 1993 *Acta Phys. Pol.* B **24** 1301
[13] Koma Y, Suganuma H and Toki H 1999 *Phys. Rev.* D **60** 074024
[14] Bali G S 1998 *Preprint* hep-ph/9809351 (Talk given at *3rd Int. Conf. on Quark Confinement and the Hadron Spectrum—Confinement III*)
[15] Kalb M and Ramond P 1974 *Phys. Rev.* D **9** 2273
Goddard P, Goldstone J, Rebbi C and Thorn C B 1973 *Nucl. Phys.* B **56** 109
[16] Dobrowolski T 1997 *J. Phys. A: Math. Gen.* **30** 1191
Dobrowolski T 1994 *Phys. Rev.* D **50** 6503
[17] Sasaki S, Suganuma H and Toki H 1995 *Prog. Theor. Phys.* **94** 373
Suzuki T 1988 *Prog. Theor. Phys.* **80** 929

[18] Gubarev F V, Ilgenfritz E M, Polikarpov M I and Suzuki T 1999 *Phys. Lett.* B **468** 134
[19] Liu F and Mazenko G F 1992 *Phys. Rev.* B **46** 5963
Weinrib A and Halperin B I 1982 *Phys. Rev.* B **26** 1362
Halperin B 1981 *Physics of Defects* ed R Balian, M Kleman and J P Poirier (New York: North-Holland)
[20] Ellis J, Heinz U and Kowalski H 1989 *Phys. Lett.* B **233** 223
[21] Bjorken J D 1983 *Phys. Rev.* D **27** 140